Comparative Study of Resonant Frequency of Rectangular Microstrip Antenna

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Abstract—*This paper theoretically studies the effect of antenna dimensions Length, Width, relative permittivity and substrate thickness on the three models of resonant frequency of rectangular microstrip antenna. The calculated values of resonant frequency are compared with the measured values. In our study we have used improved design equation for calculation of effective dielectric constants and fringing length extension to evaluate resonant length and frequency. The models agree with the measured values for thin substrate at lower frequency while at higher frequency calculated values differ significantly. We optimized the models of resonant frequency using Adaptive Neuro Fuzzy Inference system implemented in Matlab platform. We have also performed tolerance analysis of fractional change in resonant frequency with antenna dimensions namely length, substrate thickness and dielectric constant. The fractional change in resonant frequency is found to be within 1%-1.38% for thin substrate and 1.5% to 2.73% for moderate thickness. However for thicker substrate changes are significant.*

Keywords*: Effective dielectric constant, Fractional change in resonant frequency, Fringing length extension, Substrate height, Tolerance analysis*

1. INTRODUCTION

The conventional formulas used in the design of the microstrip patch antenna differ significantly from improved design formulas as a result calculated dimensions failed to resonate the antenna at a desired frequency of operation. Also due to considerable error in calculated resonant length results in large variation in fractional change in resonant frequency beyond tolerance limit.

1.1 Related works

In the year 1996, M. Kara [1] derived three close-form expressions for resonant frequencies of rectangular microstrip antenna with thick substrates using modified transmission-line and cavity model. His design formulas are in excellent agreement with measurements.

In the year 1998, P. Mythili et al [2], presented a simple approach to determine the resonant frequencies of regular shaped microstrip antennas and compared the results with the experimental results

In the year 2007, K. Guney et al [3] studied the resonant frequency of rectangular microstrip antennas with thin and thick substrates using adaptive Neuro-fuzzy Inference System (ANFIS).

In 2011, Ali Akdagli et al [4], presents an expression for resonant length to calculate the resonant frequency of C-shape compact microstrip antennas operating on UHF band.

In the year 2013, Guru Pyari Jangid et al [5] studied resonant frequency of rectangular microstrip antenna using Artificial Neural Networks (ANNs) with two stage training.

The goal of this paper is to study the resonant of the antenna proposed by Sengupta [6], James et al [7] and Hammersted [8] using improved design formula and compare the result with measured values. To optimize the resonant frequency we use Adaptive Neuro-Fuzzy Inference System (ANFIS), implemented in Matlab platform to perform simulation. Then, we study effect of tolerance in patch length on resonant frequency.

2. DESIGN CONSIDERATIONS

The accurate calculation of effective dielectric constant is an important consideration to account for fringing length extension and effective length as it is related to resonant frequency [9]. For simulation and calculation of effective dielectric constant, the patch metallization thickness is taken as **17.78**$\boldsymbol{\mu m}$ (**0.5 oz** copper foil) and substrate height in the range $\mathbf{0.17\ mm} \le \boldsymbol{h} \le \boldsymbol{9.25\ mm}$. The dielectric constant of the substrate $\boldsymbol{\varepsilon_r}$ is taken in the range of $\mathbf{2.22} \le \boldsymbol{\varepsilon_r} \le \mathbf{10.2}$ for generation of random data to optimize length and width and the same optimized values are used to optimization of resonant frequency of three models in the frequency interval of **1.0-2.4 GHz**. The testing data used for simulation are the measured values taken from various journals [3]-[5].

3. THEORITICAL FORMULATIONS

A rectangular microstrip antenna can be represented by a metalized patch of width **W** and length **L** printed on a substrate of thickness **h**, and relative dielectric constant $\boldsymbol{\varepsilon_r}$ with a metalized ground plane at the bottom having dimension greater than that of the patch. The resonant frequency of different models proposed by Sengupta [6], James et al [7] and Hammersted [8] for the rectangular printed antenna can be calculated by conventional expression of effective dielectric constant and fringing length extensions or improved design equation accounting patch metallization thickness. Initially we use conventional design formulas to calculate relative error and finally we use improved expressions for simulation and optimization.

The resonant frequency model proposed by Sengupta can be written as [6]**:**

$$f_r = f_0\left[1-\frac{2h}{\varepsilon_{ef}(W)L\pi\alpha}\right] / \left(1+\frac{2h}{\varepsilon_{ef}(W)L\pi\alpha}\ln\left(2L\sqrt{\varepsilon_{ef}(W)}\frac{1}{\gamma h}\right)\right) \tag{1}$$

$$\text{Where } f_0 = \frac{c}{2L\sqrt{\epsilon_{ef(W)}}} \tag{1a}$$

With $\alpha = 1+1.393\frac{h}{W}+0.667\frac{h}{W}\ln(\frac{h}{W}+1.44), \gamma = 1.78107$ and

$$\varepsilon_{ef}(W) = 0.5(\varepsilon_r+1)+0.5(\varepsilon_r-1)/\sqrt{[1+10h/W]}$$

The resonant frequency model proposed by James et al can be expressed as [7]:

$$f_r = f_0\varepsilon_r/\sqrt{\varepsilon_{ef}(W)\,\varepsilon_{ef}(L)}\,(1+\delta) \tag{2}$$

Where $\delta = \frac{h}{W}\{0.882+\left[\frac{0.164(\varepsilon_r-1)}{\varepsilon_r^2}\right]+(\varepsilon_r+1)/\varepsilon_r\pi\left[0.758+\ln\left(\frac{W}{h}+1.88\right)\right]\}$,

$\varepsilon_{ef}(W)$ is effective dielectric constants defined earlier and

$$\varepsilon_{ef}(L)=0.5(\varepsilon_r+1)+0.5(\varepsilon_r-1)/\sqrt{[1+10h/L]} \tag{2a}$$

The resonant frequency model proposed by Hammersted can be expressed as [8]:

$$f_r = f_0/(1+\Delta L/L) \tag{3}$$

Where ΔL is fringing length extension and in terms of the conventional formula it can be expressed as [9]:

$$\Delta L = 0.412h\frac{(\varepsilon_{ef}+0.3)\left({}^{W}\!/_{h}+0.264\right)}{(\varepsilon_{ef}-0.258)\left({}^{W}\!/_{h}+0.8\right)} \tag{4}$$

Where ε_{ef} is the effective dielectric constant and the most commonly used design formula is can be written as [9]:

$$\epsilon_{ef}=0.5(\varepsilon_r+1)+0.5(\varepsilon_r-1)/\sqrt{[1+12h/W]} \tag{5}$$

The improved expression of ε_{ef} can be expressed as [11]

$$\varepsilon_{ef} = 0.5(\epsilon_r+1)+0.5(\epsilon_r-1)/G \tag{5a}$$

Where $G = \left[1+\frac{10}{u}\right]^{ab}-\frac{ln4}{\pi}\frac{t}{\sqrt{Wh}}$, u=W/h, a=1+$\frac{1}{49}\ln\left[\frac{u^4+\left(\frac{u}{52}\right)^2}{u^4+0.432}\right]+\frac{1}{18.7}\ln[1+\ln\{1+(u/18.1)^3\}]$,and b=0.564$exp[-0.2/(\varepsilon_r+0.3)]$

The improved expression of fringing length extension can be expressed as [10]:

$$\Delta L = hE_1E_3E_5/E_4 \tag{6}$$

where $E_1 = 0.434907\frac{\varepsilon_{ef}^{0.81}+0.26}{\varepsilon_{ef}^{0.81}-0.189}\frac{(u)^{0.8544}+0.236}{(u)^{0.8544}+0.87}$, $E_2 = 1+\frac{(u)^{0.371}}{2.358\varepsilon_r+1}$,

E_3 =1+$0.5274arctan\left[0.084(u)^{1.9413/E_2}\right]/\varepsilon_{ef}^{0.9236}$

$E_4 = 1+0.0377arctan\left[0.067(u)^{1.457}\right]\{6-5exp[0.036(1-\varepsilon_r)]\}$,

$E_5 = 1-0.218exp(-7.5u)$ and u= W/h

The expression for patch length and width is given as [9]:

$$L = c/2f\sqrt{\varepsilon_{ef}}-2\Delta L \tag{7}$$

$$W = c/2f\sqrt{2/(\varepsilon_r+1)} \tag{8}$$

3.1 Adaptive Neuro-Fuzzy Inference System (ANFIS)

The ANFIS is a hybrid learning algorithm which identifies the membership function parameters of single-output, Sugeno type fuzzy inference systems (FIS). A combination of least-squares and back propagation gradient descent methods are used for training FIS membership function parameters to model a given set of input to output data. Error is the array of root mean square training errors (difference between the FIS output and the training data output) at each epoch. The ANFIS architecture has five layers namely fuzzy layer, product layer, normalized layer, de-fuzzy layer, and output layer

4. SIMULATION

In first part of simulation of ANFIS model we provide the three inputs as resonant frequency (f_r), substrate height (h) and dielectric constant ($\boldsymbol{\varepsilon_r}$) and get optimized width (**W**) as outputs the membership values were specified such that the total number of rules is 36. In the second stage in addition to all the three inputs of first stage we use the fourth parameter as the optimized width inputs and get optimized length (**L**) and the membership values are specified as (**3 3 2 3**) the total number of rules is 54. We have used the measured data of [5] for testing. And finally the optimized **W** and **L** are used to optimize three resonant frequency models proposed by [6]-[8] for comparative study, the same rules are applied here**.** Entire simulation is performed in MATLAB platform by interfacing Adaptive Neuro-Fuzzy Inference System (ANFIS) technique. The improved design equations namely **Eqn.5a** to **Eqn.8** are used to generate training data sets and measured data are used as testing data. In the ANFIS model three similar design stages are formulated to optimize resonant frequency. Initially we have used product sigma membership function which a product of two sigmoid membership functions to train the network.

5. RESULTS AND DISCUSSIONS

The five different substrate materials are used in the entire simulations, the calculated and measured values of antenna dimension and resonant frequency are given in **Table-I** and **II**. It is seen that the calculated values almost agree with measured resonant frequency for three models but the measured patch width differ significantly. Since measured dimension and resonant frequency values differ so we carry out optimization with eight data sets to train the ANFIS model. The optimized values of width, length and three resonant frequency models are listed in **Table-III**. It is seen that the optimized values of resonant frequency are in good agreement with the observed frequency for thin substrate. While for thicker substrate with high dielectric constant, resonant frequency values differ slightly. In the first stage of training three variables are used to optimize width of the patch and the root mean square error (**RMSE**) is **0.0117** at epoch number **300**. The optimized patch width is then used in second stage of ANFIS model produced an optimized patch length resulting in an **RMSE** error of **0.0021**. The **RMSE** error for third, fourth and fifth optimization are **2.7730**, **2.7755** and **2.8030** respectively. The normalized resonant frequency and relative errors as function of observed frequency is simulated and optimized for three resonant frequency models and responses are shown in **Fig.1-6.** Comparisons of fractional change in resonant frequency of models against observed frequency are shown in **Fig.7-8**. It is observed that the resonant frequency model proposed by Hammersted gives nominal error$(\in\leq \mathbf{1\%})$ and$(\in\leq \mathbf{2.5\%})$ for thin and thick substrate respectively. While James and Sengupta's model can predict resonant frequency for substrate with thin and moderate height within **2%** to **5.2%** of error**.**

Table I: Comparison of three resonant frequency models with measured values

Calculated frequency (GHz) Eqn.(1a)	ε_r	h mm	Calculated		Resonant frequency of different models (GHz)			
			W (mm)	L (mm)	Eqn.(1)	Eqn.(2)	Eqn.(3)	Measured [3]-[5]
7.847	2.22	0.170	15.30	12.94	7.678	7.597	7.728	7.740
3.182	2.33	3.175	40.22	32.21	2.889	2.701	2.904	2.890
1.374	2.50	1.524	84.37	70.03	1.334	1.312	1.340	1.344
7.355	2.55	1.630	17.16	13.43	6.636	6.105	6.593	6.560
4.960	4.35	0.790	19.39	15.03	4.782	4.601	4.671	4.730
5.092	10.2	1.270	13.78	9.95	4.922	4.466	4.482	4.600

Table II: Tolerance analysis of rectangular microstrip antenna thin and thick substrate.

ε_r	h (mm)	W (mm)	Effect of length difference on resonant frequency with existing and improved design equations at 2.4 GHz							
			ΔL mm	Δf_r MHz (Eqn.1)	% error	Δf_r MHz (Eqn.2)	% error	Δf_r MHz (Eqn.3)	% error	$\Delta f_r/f_r$ %
2.22	3.175	49.26	1.10	58.5	2.4	64.8	2.72	34.5	1.41	2.72
2.22	9.25	49.26	1.79	112.8	4.24	127.1	5.20	62.5	2.43	4.97
2.5	1.58	47.25	0.51	31.6	1.31	32.1	1.35	17.1	0.71	1.58
2.5	6.26	47.25	1.47	89.3	3.50	103.3	4.26	52.1	2.09	4.08
4.35	1.27	38.21	0.23	20.9	0.86	19.4	0.80	10.4	0.44	1.23
4.35	3.30	38.21	0.57	41.6	1.64	50.7	2.08	26.4	1.10	2.16
10.2	0.170	26.41	0.09	16.1	0.70	11.0	0.46	10.6	0.44	1.05
10.2	1.63	26.41	0.08	5.4	0.21	10.7	0.44	7.3	0.31	1.18

Table III: Optimized resonant frequency & fractional change for thin & thick substrate at 2.4 GHz

Dimensions of h, Optimized W & L in mm				Optimized resonant frequency of Eqn.1-3 at 2.4 GHz & fractional in frequency with observed values					
ε_r	h	W	L	$f^{Opt}_{Eqn.1}$	% error	$f^{Opt}_{Eqn.}$	% error	$f^{Opt}_{Eqn.3}$	% error
2.22	3.175	49.26	40.15	2.383	0.72	2.318	0.72	2.408	3.40
2.22	9.52	49.24	33.53	2.660	2.660	2.441	2.441	2.575	2.575
2.33	1.628	48.43	39.16	2.436	1.52	2.407	1.52	2.457	0.29
2.33	8.54	48.45	33.56	2.634	2.634	2.439	2.440	2.548	2.548
2.50	1.58	47.25	38.87	2.382	0.74	2.350	0.75	2.391	1.70
2.5	6.26	47.24	34.3	2.561	2.561	2.426	2.426	2.492	2.492

4.35	1.27	38.22	29.49	2.440	1.65	2.360	1.66	2.383	1.65
4.35	3.3	38.22	28.28	2.526	2.526	2.440	2.441	2.395	2.396
10.2	0.170	26.41	19.50	2.425	1.04	2.397	1.04	2.399	0.12
10.2	1.63	26.41	19.27	2.513	2.514	2.445	2.446	2.350	2.351

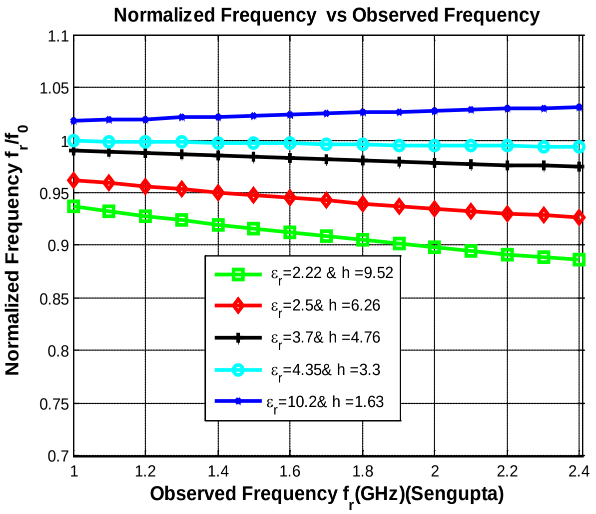

Fig. 3: ANFIS normalized resonant frequency as a function of frequency(Eqn.1)

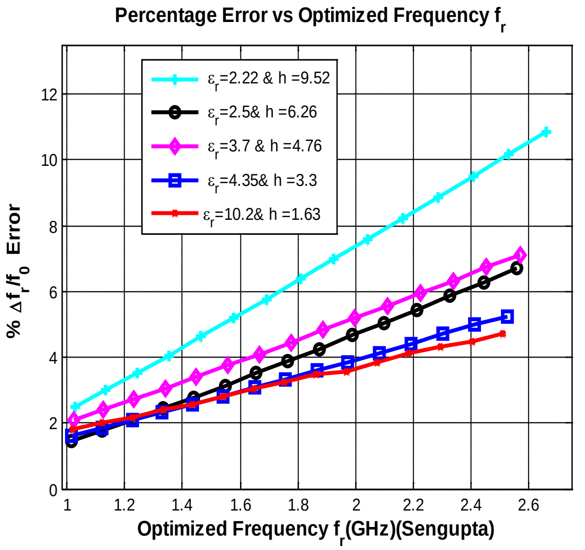

Fig. 4: Fractional change in resonant frequency as a function of optimized frequency (Eqn.1)

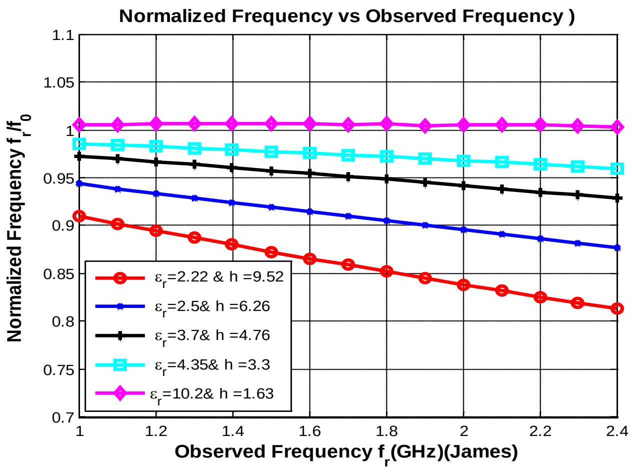

Fig. 5: ANFIS normalized resonant frequency as a function of frequency (Eqn.2)

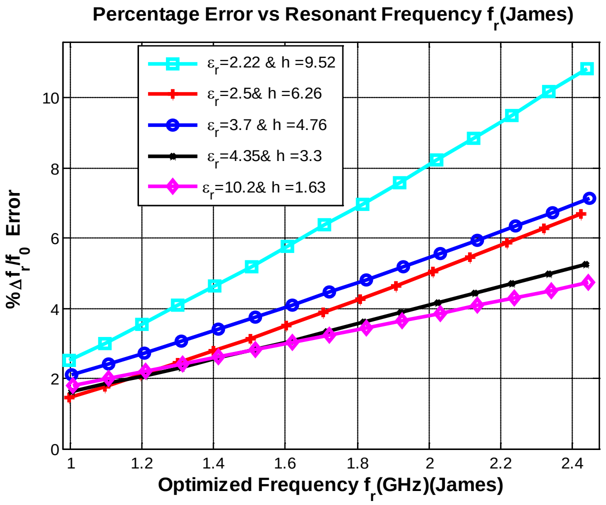

Fig. 6: Fractional change in f_r as a function of optimized frequency (Eqn.2)

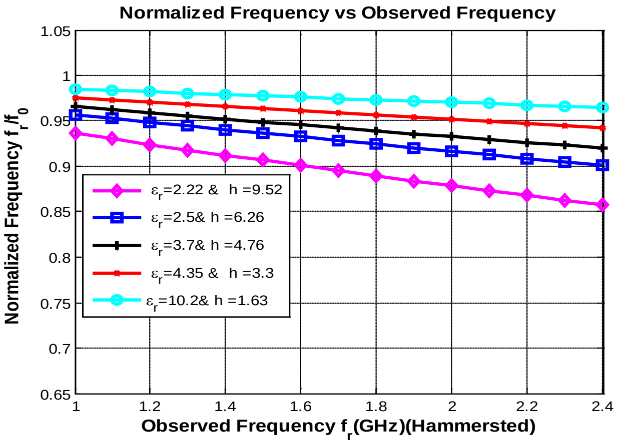

Fig. 7: ANFIS normalized resonant frequency as a function of frequency (Eqn.3)

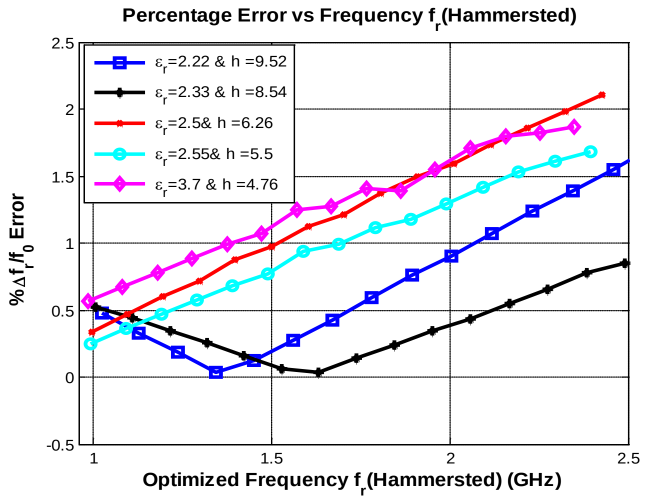

Fig. 8: Fractional change in resonant frequency as a function of optimized frequency (Eqn.3)

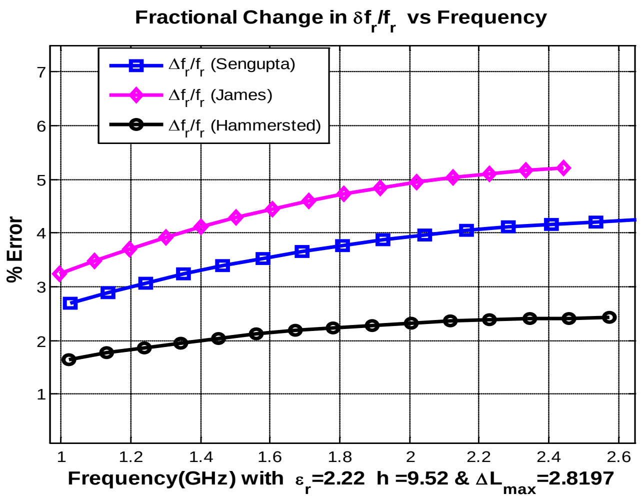

Fig. 9: Comparison of fractional change in resonant frequency (Eqn.1-3) for thick substrate.

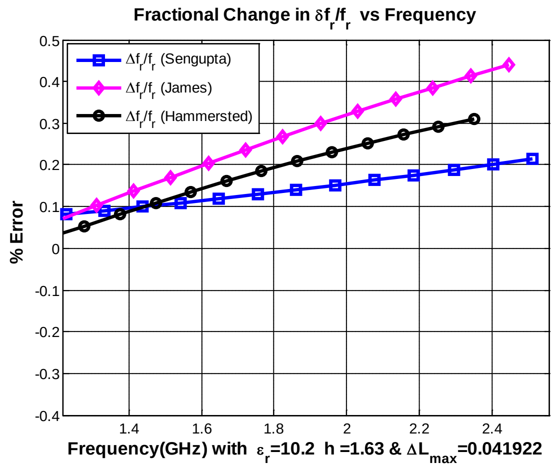

Fig. 10: Comparison of fractional change in resonant frequency (Eqn.1-3) for thin substrate.

6. CONCLUSIONS

Theoretical and simulated analysis of three resonant frequency models for rectangular microstrip antenna with eight dielectric substrates is performed with improved design equations. In the entire optimization process including patch width, length and three resonant frequency models calculation total time is approximately 675 seconds. The parameters of the patch antenna are optimized by using adaptive Neuro-fuzzy inference optimization technique and the operating frequency is 2.4 GHz. Through our simulated analysis it may be concluded that improved design equations are more accurate for calculation of antenna dimensions and resonant frequency. The maximum value fractional change in resonant frequency occurs for thick substrate. The resonant frequency model of Hammersted is least sensitive to improved design equations while James and Sengupta's formulas can accurately predict resonant frequency with minimum error when improved design equations are used

7. ACKNOWLEDGMENT

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